Finite Volume/Immersed Boundary Solvers for Compressible Flows: Development and Applications

A thesis submitted in partial fulfillment of the requirements for the Degree of

DOCTOR OF PHILOSOPHY

in Mechanical Engineering by

Shuvayan Brahmachary



Department of Mechanical Engineering Indian Institute of Technology Guwahati Guwahati - 781 039 "The journey is the reward" — The Pirates (Steve Jobs)

Dedicated to Ma, Baba and Dada

CERTIFICATE

This is to certify that the work presented in the thesis entitled "Finite Volume/Immersed Boundary Solvers for Compressible Flows: Development and Applications" submitted by Shuvayan Brahmachary to Indian Institute of Technology Guwahati for the award of the degree of Doctor of Philosophy in Mechanical Engineering is a bona fide record of research work carried out by the student himself under our supervision and have not been submitted elsewhere for any degree or diploma.

Signature:

Date:

Supervisor: Dr. Ganesh Natarajan Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati-781039, Assam, India.

Signature:

Date:

Supervisor: Prof. Niranjan Sahoo Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati-781039, Assam, India.

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ABSTRACT

This thesis is devoted to the development of a robust and accurate Immersed Boundary/Finite Volume (IB-FV) framework for compressible flows and their applications to design and optimisation. The framework is devised by combining an unstructured data based finite volume flow solver with a sharp interface immersed boundary method. The finite volume flow solver employs limited linear reconstruction in conjunction with vanLeer and AUSM schemes for convective fluxes while central differencing is employed for viscous fluxes. A new approach to compute gradients, which are critical to the computation of inviscid and viscous fluxes, based on a variant of Gauss divergence theorem is proposed. The strategy referred to as Modified Green Gauss (MGG) reconstruction is a one-step approach but leads to marginally lesser dissipation and allows for the use of marginally higher Courant number than existing reconstruction techniques. A novel non-iterative variant of MGG reconstruction for non-orthogonal meshes is also described and its robustness in high-speed flows has been studied. A sharp-interface Immersed Boundary (IB) technique based on local reconstruction of the solution has been proposed for inviscid and viscous flows. The boundary conditions are imposed directly at the geometry interface and is employed to obtain the solution in the near vicinity of the solid(s). This reconstruction approach which also employs the finite volume solutions obtained away from the solid, is effectively an interpolation technique that does not strictly conserve the mass, momentum and energy. Two different strategies, based on inverse distance weighting (IDW) for inviscid flows and one-dimensional reconstruction (HCIB) for viscous flows are described and explained in this work. We show that the finite mass conservation errors diminish linearly with grid refinement and that the reconstruction approach does not degrade the nominal second-order accuracy of the flow solver. The IB-FV solver computes wall pressure and skin-friction distributions quite accurately, although the latter requires sufficient fine meshes in the vicinity of the body. However, finite levels of mesh refinement does not produce accurate heat flux estimates in laminar hypersonic flows past blunt geometries. We probe the possible causes of this under-prediction using an in-depth diagnostic analysis. The investigations indicate that errors due to temperature reconstruction which are linked to a loss in energy conservation are primarily responsible for the inaccurate estimation of wall heat-flux and stagnation point heat transfer. We prove using numerical experiments that the use of adaptive meshes and non-linear/non-polynomial interpolations do not improve the heat flux estimates and that the errors are larger as Reynolds and Mach numbers become higher. The utility of the FV and IB-FV frameworks proposed in this work are highlighted by their application to three selected problems of design and optimisation. These frameworks are employed in conjunction with variable fidelity approaches for the design of minimum drag geometries, scramjet intakes and supersonic nozzle. The large spectrum of canonical problems in this thesis over a wide range of Mach and Reynolds number indicate the efficacy of the IB-FV solver while also highlighting some of its drawbacks. The IB-FV framework, despite its limitations, is also found to be a promising tool to evolve multi-fidelity optimisation frameworks that can accelerate the design and optimisation in hypersonic flows.

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CHAPTER 1

INTRODUCTION

"Heavier-than-air flying machines are impossible"

- Lord Kelvin, 1895, On sustainable flight

"Houston, Tranquility Base here. The Eagle has landed"

- Neil Armstrong, 1969, On moon landing

H igh speed flows has been a subject matter of great interest to various groups of researchers as well as commoners. The unwavering desire to mimic bird's flight has propelled businessman turned inventors the Wright brothers, into discovering the "flying-machine" in 1903, which since then has seen tremendous alterations by scientist, in arriving at today's interplanetary flight vehicles [1]. The remarkable leap of such structural design in the last century has foreseen many hurdles which were primarily caused by the highly complex flow features, one of which include the infamous crash of Ralph Virden P-38 aircraft in 1941. This incident was attributed to the "compressibility effects" which was later addressed by NACA Langley Memorial Laboratory and Ames Aeronautical Laboratory. What followed this unfortunate incident was the re-embarkment of humankind into a new flow regime of supersonic flow where the so-called "Mach barrier" was crossed by Chuck Yeager in 1947 in his Bell X-1. This was soon followed by the first hypersonic flight by Robert White at Mach number of 5.3 in his X-15.

This remarkable feat was made possible by countless hours of research carried out by scientist to generate highly precise experimental data and empirical correlations.